

# FuREM - *A System for Visualization and Manipulation of L-Fuzzy Relations*

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RELVIEW is a convenient tool to visualize and manipulate finite relations. Such a tool can be very helpful when working with relations either theoretical or in applications.

FUREM (**F**uzzy **R**elation **M**anipulator) is supposed to be a similar system that is capable of handling so-called  $L$ -fuzzy relations.



For the definition of Dedekind and arrow categories I refer to my presentations on Wednesday.

We call a Heyting algebra with a binary operation  $*$  that is associative, commutative, has the greatest element of  $L$  as a neutral element, and is continuous with respect to join in both parameters, a fuzzy algebra.

## Definition

A fuzzy category  $\mathcal{F}$  is an arrow category together with two operations  $*$  and  $\circ$ ; so that the following holds:

- 1  $*$  maps two relations  $Q : A \rightarrow B$  and  $R : A \rightarrow B$  to a relation  $Q * R : A \rightarrow B$ .
- 2  $*$  is associative, commutative and continuous.
- 3  $Q * R^\downarrow = Q \sqcap R^\downarrow$  for all  $Q, R : A \rightarrow B$ .
- 4  $(Q * R)^\sim = Q^\sim * R^\sim$  for all  $Q, R : A \rightarrow B$ .



## Definition

- 5  $\ast$  maps two relations  $Q : A \rightarrow B$  and  $R : B \rightarrow C$  to a relation  $Q \ast R : A \rightarrow C$ .
- 6  $\ast$  is associative and continuous.
- 7  $Q \ast R^\downarrow = Q; R^\downarrow$  for all  $Q : A \rightarrow B$  and  $R : B \rightarrow C$ .
- 8  $(Q \ast R)^\sim = R^\sim \ast Q^\sim$  for all  $Q : A \rightarrow B$  and  $R : B \rightarrow C$ .
- 9 The exchange inclusion  $(Q \ast R) \ast (S \ast T) \sqsubseteq Q \ast S \ast R \ast T$  is valid for all  $Q, R : A \rightarrow B$  and  $S, T : B \rightarrow C$ .
- 10 The following versions of the modular inclusion are valid:
  - 1  $Q; R \ast S \sqsubseteq Q; (R \sqcap Q^\sim \ast S)$ ,
  - 2  $Q \ast R \ast S \sqsubseteq Q; (R \ast Q^\sim \ast S)$ ,
  - 3  $(P \ast Q) \ast R \ast S \sqsubseteq P \ast (R \ast Q^\sim \ast S)$ ,
 for all  $P, Q : A \rightarrow B$ ,  $R : B \rightarrow C$ , and  $S : A \rightarrow C$ .



### Definition

A hive  $\mathcal{H}$  (of fuzzy categories) is a family of fuzzy categories that share the same subcategory of crisp relations. A hive is called complete (or a c-hive) iff for every fuzzy algebra  $F$  there is a fuzzy category  $\mathcal{F}$  in  $\mathcal{H}$  so that  $F$  is isomorphic to  $\text{Sc}(\mathcal{F})$ .

If  $F$  is a fuzzy algebra, then we denote by  $\text{Rel}_{\text{fin}}(L)$  the fuzzy category of finite sets with  $L$ -relations between them.



- 1 The FuREM system uses the hive of fuzzy categories  $\text{Rel}_{\text{fin}}(F)$  for all finite fuzzy algebras  $F$ .
- 2 The system allows to define (and/or load) and use any finite fuzzy algebra.
- 3 The system allows to define (and/or load) objects, i.e., finite sets with named elements. Regular operations such as product, disjoint unions, and fuzzy power sets can be applied to objects.
- 4 The system allows to define (and/or load) relations between objects in a specific fuzzy category of kind  $\text{Rel}_{\text{fin}}(F)$  where  $F$  is a finite fuzzy algebra.
- 5 FuREM is capable of computing relational expressions based on the language of fuzzy categories with relation products, relational sums, splittings of crisp partial equivalence relations and fuzzy powers. Relational expressions are strongly typed and type checking (resp. type inference) is done at compile time.



- ⑥ FuREMuses algebraic decision diagrams (ADDs) to store and manipulate relations similar to the usage of binary decision diagrams (BDDs) in RELVIEW.
  - In comparison to the RELVIEW system, the FuREM system is an extension. It allows to work with arbitrary  $L$ -fuzzy relations instead of Boolean valued relations only.
  - Since there are several specific implementation details/tricks that are restricted to BDDs and cannot be generalized easily to ADDs, the RELVIEW system is, even in the case of Boolean relations, faster than the current implementation of FuREM.



The FuReM system is written in Java and utilizes the following external packages/libraries:

- CUDD. The CUDD package is a package written in C for the manipulation of decision diagrams. It supports binary decision diagrams (BDDs), algebraic decision diagrams (ADDs), and Zero-Suppressed BDDs (ZDDs).
- JParsec 3.0. Jparsec is a recursive-descent parser combinator framework written for Java. It's an implementation of Haskell Parsec on the Java platform.
- LatDraw. Java library for drawing lattices.





System demonstration.

There is some work left to be done before the first version of FuREM will become available (probably Spring/Sommer 2022).

- 1 Saving of components.
- 2 Create manipulators for creating new fuzzy algebras and set objects.
- 3 Edit programs in system.
- 4 Port the system to Linux/Unix and Mac OS.
- 5 Test the system intensively on all platforms.
- 6 Documentation and user manual.



- 1 L<sup>A</sup>T<sub>E</sub>X output of relations.
- 2 Replace CUDD by an implementation of generalized ADDs.
- 3 Add a small functional programming language on top of the relational expressions.



Thank you for your attention.

