Relational Computation of Sets of Relations

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Introduction

The use of ROBDDs often leads to an amazing computational power of RelView, in particular, if the solution of a hard problem is based on the computation of a subset $\mathcal{R}$ of a powerset 2^X.

In certain situations X is a direct product $Y \times Z$, which means that $\mathcal{R}$ is a subset of $[Y \leftrightarrow Z]$, i.e., a set of relations.

Examples:

- Solutions of a timetabling problem (B. Kehden, R.B).
- Computation of the set of up-closed multirelations on a given set (W. Guttmann).
- Computational proof of a variant of the Kuratowski closure-complement-theorem (R. B.).
- Experiments with sufficient criteria for the existence of kernels (R. B.).

We present a general technique for the relational computation of sets $\mathcal{R}$ of relations.

It is based on a specification of a relation R to belong to $\mathcal{R}$ by means of an inclusion $\mathfrak{s} \subseteq \mathfrak{t}$, where $\mathfrak{s}$ and $\mathfrak{t}$ are **relation-algebraic expressions constructed** from the vector model $s := \mathrm{vec}(R)$ of R in a specific way.

To get the specification $\mathfrak{s} \subseteq \mathfrak{t}$, we frequently apply properties of the mapping vec (last talk) and its inverse mapping Rel.

The desired computation of $\mathcal{R}$ via a relation-algebraic expression $\mathfrak{r}$ is then obtained from $\mathfrak{s} \subseteq \mathfrak{t}$ in **one step**.

Compared with the direct development of $\mathfrak{r}$ from a logical specification of the relation R to belong to $\mathcal{R}$, the proposed **technique is much simpler**.

Column-wise Extendible Vector Expressions

Decisive for the correctness of the step from $\mathfrak{s} \subseteq \mathfrak{t}$ to $\mathfrak{r}$ is the **syntactic structure** of both sides of the inclusion.

Definition: For $s : X \leftrightarrow \mathbf{1}$, the set $\mathfrak{V}(s)$ of **typed column-wise extendible vector expression** over s is inductively defined as follows:

- We have $s \in \mathfrak{V}(s)$ and its type is $X \leftrightarrow \mathbf{1}$.
- If $v : Y \leftrightarrow \mathbf{1}$, then $v \in \mathfrak{V}(s)$ and its type is $Y \leftrightarrow \mathbf{1}$.
- If $\mathfrak{t} \in \mathfrak{V}(s)$ has type $Y \leftrightarrow \mathbf{1}$, then $\overline{\mathfrak{t}} \in \mathfrak{V}(s)$ and its type is $Y \leftrightarrow \mathbf{1}$.
- If $\mathfrak{t}, \mathfrak{u} \in \mathfrak{V}(s)$ have type $Y \leftrightarrow \mathbf{1}$, then $\mathfrak{t} \cup \mathfrak{u} \in \mathfrak{V}(s)$, $\mathfrak{t} \cap \mathfrak{u} \in \mathfrak{V}(s)$ and their types are $Y \leftrightarrow \mathbf{1}$.
- If $\mathfrak{t} \in \mathfrak{V}(s)$ has type $Y \leftrightarrow \mathbf{1}$ and $\mathfrak{R}$ is a relation-algebraic expression of type $Z \leftrightarrow Y$ free of s, then $\mathfrak{R};\mathfrak{t} \in \mathfrak{V}(s)$ and its type is $Z \leftrightarrow \mathbf{1}$.

Examples:

$$s \qquad R^{\mathsf{T}};(S;s \cup v) \qquad [\![s, R;s]\!]$$

In a column-wise extendible vector expression over s the vector s can be seen as a variable in the logical sense. Using this interpretation, next we define how to replace s by a relation of appropriate type.

Definition: For $s : X \leftrightarrow \mathbf{1}$, $\mathfrak{s} \in \mathfrak{V}(s)$ and $R : X \leftrightarrow Z$ we define $\mathfrak{s}[R/s]$ of type $X \leftrightarrow Z$ inductively as follows:

- $s[R/s] = R$.
- $v[R/s] = v;\mathsf{L}$, with $\mathsf{L} : \mathbf{1} \leftrightarrow Z$.
- $\overline{\mathfrak{t}}[R/s] = \overline{\mathfrak{t}[R/s]}$,
- $(\mathfrak{t} \cup \mathfrak{u})[R/s] = \mathfrak{t}[R/s] \cup \mathfrak{u}[R/s]$ and $(\mathfrak{t} \cap \mathfrak{u})[R/s] = \mathfrak{t}[R/s] \cap \mathfrak{u}[R/s]$.
- $(\mathfrak{R};\mathfrak{t})[R/s] = \mathfrak{R};(\mathfrak{t}[R/s])$.

Examples:

$$\begin{aligned} s[\mathsf{M}/s] &= \mathsf{M} \\ R^{\mathsf{T}};(S;s \cup v)[\mathsf{M}/s] &= R^{\mathsf{T}};(S;\mathsf{M} \cup v;\mathsf{L}) \\ [\![s, R;s]\!][\mathsf{M}/s] &= [\![\mathsf{M}, R;\mathsf{M}]\!] \end{aligned}$$

Recall the mapping

$$\mathrm{vec} : [X \leftrightarrow Y] \to [X \times Y \leftrightarrow \mathbf{1}] \qquad \mathrm{vec}(R) = [\![R, \mathsf{I}]\!];\mathsf{L}.$$

Together with the mapping (with π and ρ as projection relations of $X \times Y$)

$$\mathrm{Rel} : [X \times Y \leftrightarrow \mathbf{1}] \to [X \leftrightarrow Y] \qquad \mathrm{Rel}(s) = \pi^{\mathsf{T}};(s;\mathsf{L} \cap \rho)$$

it forms a Boolean lattice isomprphism between $[X \leftrightarrow Y]$ and $[X \times Y \leftrightarrow \mathbf{1}]$.

Theorem 1. Assume the subset $\mathcal{R}$ of $[X \leftrightarrow Y]$ to be specified as

$$\mathcal{R} = \{\mathrm{Rel}(s) \mid s : X \times Y \leftrightarrow \mathbf{1} \wedge \mathfrak{s} \subseteq \mathfrak{t}\},$$

where $\mathfrak{s}, \mathfrak{t} \in \mathfrak{V}(s)$ have type $Y \leftrightarrow \mathbf{1}$. By means of $\mathbf{M} : X \times Y \leftrightarrow [X \leftrightarrow Y]$ and $\mathsf{L} : \mathbf{1} \leftrightarrow Y$ we get a vector $\mathfrak{r} : [X \leftrightarrow Y] \leftrightarrow \mathbf{1}$ that models the set $\mathcal{R}$ as

$$\mathfrak{r} := \overline{\mathsf{L};(\mathfrak{s}[\mathbf{M}/s] \cap \overline{\mathfrak{t}[\mathbf{M}/s]})}^{\mathsf{T}}.$$

Theorem 1 is a specialisation of a more general theorem of R. B. (MPC 2015) to the case that $\mathcal{R}$ is a set of relations.

Given the subset $\mathcal{R}$ of $[X \leftrightarrow Y]$, the development of a vector

$$\mathfrak{r} : [X \leftrightarrow Y] \leftrightarrow \mathbf{1}$$

that models the set $\mathcal{R}$ consists of **three steps**:

1. Take an arbitrare vector $s : X \times Y \leftrightarrow \mathbf{1}$.
2. Develop an inclusion $\mathfrak{s} \subseteq \mathfrak{t}$ such that:

$$\mathfrak{s} \in \mathfrak{V}(s) \qquad \mathfrak{t} \in \mathfrak{V}(s) \qquad \mathrm{Rel}(s) \in \mathcal{R} \Longleftrightarrow \mathfrak{s} \subseteq \mathfrak{t}.$$

3. Define:

$$\mathfrak{r} := \overline{\mathsf{L};(\mathfrak{s}[\mathbf{M}/s] \cap \overline{\mathfrak{t}[\mathbf{M}/s]})}^{\mathsf{T}}.$$

Helpful Properties for the Second Step

Theorem 2. (B. Kehden) For all Q, R and S we have

$$\mathrm{vec}(Q;R;S) = (Q \parallel S^{\mathsf{T}});\mathrm{vec}(R)$$

Theorem 3. (M. Winter) Assume $R : X \leftrightarrow X$ and $S : X \leftrightarrow X$ and let π and ρ be the projection relations of X^2 and α and β those of $X^2 \times X^2$. Define $C : X^2 \times X^2 \leftrightarrow X^2$ as

$$C := (\mathsf{I} \cap \alpha;\rho;\pi^{\mathsf{T}};\beta^{\mathsf{T}});(\pi \parallel \rho).$$

Then we have

$$\mathrm{vec}(R;S) = C^{\mathsf{T}};[\![\mathrm{vec}(R), \mathrm{vec}(S)]\!].$$

Theorem 4. Assume $R : X \leftrightarrow X$ and let π and ρ be the projection relations of X^2. Then we have

$$\mathrm{vec}(R^{\mathsf{T}}) = [\rho, \pi]\!];\mathrm{vec}(R).$$

Example: Transitive Relations

Assume $s : X^2 \leftrightarrow \mathbf{1}$ to be given. With $C : X^2 \times X^2 \leftrightarrow X^2$ as introduced in Theorem 3 we have:

$$\begin{array}{lll} & \mathrm{Rel}(s) \text{ transitive} & \\ \iff & \mathrm{Rel}(s);\mathrm{Rel}(s) \subseteq \mathrm{Rel}(s) & \text{def. trans.} \\ \iff & \mathrm{vec}(\mathrm{Rel}(s);\mathrm{Rel}(s)) \subseteq \mathrm{vec}(\mathrm{Rel}(s)) & \text{isom.} \\ \iff & C^{\mathsf{T}};[\![\mathrm{vec}(\mathrm{Rel}(s)), \mathrm{vec}(\mathrm{Rel}(s))]\!] \subseteq \mathrm{vec}(\mathrm{Rel}(s)) & \text{Theorem 3} \\ \iff & C^{\mathsf{T}};[\![s, s]\!] \subseteq s & \text{isom.} \end{array}$$

As $C^{\mathsf{T}};[\![s, s]\!] \in \mathfrak{V}(s)$ and $s \in \mathfrak{V}(s)$ Theorem 1 yields

$$\boxed{\mathfrak{trans}(X) := \overline{\mathsf{L};(C^{\mathsf{T}};[\![\mathsf{M}, \mathsf{M}]\!] \cap \overline{\mathsf{M}})}^{\mathsf{T}}}$$

as relational specification of a vector $\mathfrak{trans}(X) : [X \leftrightarrow X] \leftrightarrow \mathbf{1}$ that models the set of transitive re- lations on X.

RelView-implementation of $\mathfrak{trans}(X)$, where the auxiliary program C computes the relation C:

```
C(pi,rho,alpha,beta)
  DECL A
  BEG  A = alpha*rho*pi^*beta^
       RETURN (I(A) & A)*par(pi,rho)
  END.

Trans(X)
  DECL pi, rho, alpha, beta, MM, A, L
  BEG  pi = pr1(X,X);
       rho = pr2(X,X);
       alpha = pr1(pi*pi^,pi*pi^);
       beta = pr2(pi*pi^,pi*pi^);
       MM = epsi(pi);
       A = C(pi,rho,alpha,beta)^*[|M,M];
       L = Ln1(A)
       RETURN -(L^*(A & -MM))^
  END.
```

A **direct development** (R.B., RAMiCS 2017) with π and ρ projection relations of X^2:

$$\forall x, y, z : R_{x,y} \wedge R_{y,z} \Rightarrow R_{x,z}$$

$$\iff \forall u, v : R_{u_1,u_2} \wedge R_{v_1,v_2} \wedge u_2 = v_1 \Rightarrow R_{u_1,v_2}$$

$$\iff \neg\exists\, u, v : u_2 = v_1 \wedge R_{u_1,u_2} \wedge R_{v_1,v_2} \wedge \neg R_{u_1,v_2}$$

$$\iff \neg\exists\, u, v, w :$$
$$u_2 = v_1 \wedge R_{u_1,u_2} \wedge R_{v_1,v_2} \wedge u_1 = w_1 \wedge v_2 = w_2 \wedge \neg R_{w_1,w_2}$$

$$\iff \neg\exists\, u, v :$$
$$u_2 = v_1 \wedge R_{u_1,u_2} \wedge R_{v_1,v_2} \wedge \exists\, w : (\pi;\pi^{\mathsf{T}})_{u,w} \wedge (\rho;\rho^{\mathsf{T}})_{v,w} \wedge \neg R_{w_1,w_2}$$

$$\iff \neg\exists\, u, v :$$
$$(\rho;\pi^{\mathsf{T}})_{u,v} \wedge \mathbf{M}_{u,R} \wedge \mathbf{M}_{v,R} \wedge \exists\, w : [\![\pi;\pi^{\mathsf{T}}, \rho;\rho^{\mathsf{T}}]\!]_{(u,v),w} \wedge \overline{\mathbf{M}}_{w,R}$$

$$\iff \neg\exists\, u, v :$$
$$\mathrm{vec}(\rho;\pi^{\mathsf{T}})_{(u,v)} \wedge [\![\mathbf{M}, \mathbf{M}]\!]_{(u,v),R} \wedge ([\![\pi;\pi^{\mathsf{T}}, \rho;\rho^{\mathsf{T}}]\!];\overline{\mathbf{M}})_{(u,v),R}$$

$$\iff \neg\exists\, u, v :$$
$$\mathrm{vec}(\rho;\pi^{\mathsf{T}})_{(u,v)} \wedge ([\![\mathbf{M}, \mathbf{M}]\!] \cap [\![\pi;\pi^{\mathsf{T}}, \rho;\rho^{\mathsf{T}}]\!];\overline{\mathbf{M}})_{(u,v),R}$$

$$\iff \overline{\mathrm{vec}(\rho;\pi^{\mathsf{T}})^{\mathsf{T}};([\![\mathbf{M}, \mathbf{M}]\!] \cap [\![\pi;\pi^{\mathsf{T}}, \rho;\rho^{\mathsf{T}}]\!];\overline{\mathbf{M}})}_{\perp,R}$$

A vector $\mathfrak{trans}\mathfrak{D}(X) : [X \leftrightarrow X] \leftrightarrow \mathbf{1}$ that models the set of transitive relations on X resultion from this development (with π and ρ as projection relations of X^2):

$$\mathfrak{trans}\mathfrak{D}(X) = \overline{vec(\rho;\pi^\mathsf{T})^\mathsf{T};([\![\mathbf{M},\mathbf{M}]\!] \cap [\![\pi;\pi^\mathsf{T},\rho;\rho^\mathsf{T}]\!];\overline{\mathbf{M}})}^\mathsf{T}$$

Running times of RelView-programs (PC mentioned in the last talk):

$\lvert X\rvert$	$\mathfrak{trans}(X)$	$\mathfrak{trans}\mathfrak{D}(X)$
1	0.0011	0.0007
2	0.0084	0.0040
3	0.0114	0.0166
4	0.0139	0.0172
5	0.0141	0.0264
6	0.3239	0.9037
7	259.1000	356.8000

Example: Irreflexive and Symmetric Relations

Assume $s : X^2 \leftrightarrow \mathbf{1}$ to be given. With π and ρ as projection relations of X^2 we have:

$$
\begin{array}{rcll}
\mathrm{Rel}(s) \text{ irreflexive} & \iff & \mathrm{Rel}(s) \subseteq \overline{\mathsf{I}} & \text{def. irr.} \\
& \iff & \mathrm{vec}(\mathrm{Rel}(s)) \subseteq \mathrm{vec}(\overline{\mathsf{I}}) & \text{isom.} \\
& \iff & s \subseteq \overline{\mathrm{vec}(\mathsf{I})} & \text{isom.} \\
\mathrm{Rel}(s) \text{ symmetric} & \iff & \mathrm{Rel}(s) \subseteq \mathrm{Rel}(s)^{\mathsf{T}} & \text{def. symm..} \\
& \iff & \mathrm{vec}(\mathrm{Rel}(s)) \subseteq \mathrm{vec}(\mathrm{Rel}(s)^{\mathsf{T}}) & \text{isom.} \\
& \iff & \mathrm{vec}(\mathrm{Rel}(s)) \subseteq [\rho, \pi]\!];\mathrm{vec}(\mathrm{Rel}(s)) & \text{Theorem 4} \\
& \iff & s \subseteq [\rho, \pi]\!];s & \text{isom.}
\end{array}
$$

Theorem 1 yields the following vectors $\mathfrak{irrefl}(X), \mathfrak{symm}(X) : [X \leftrightarrow X] \leftrightarrow \mathbf{1}$ that model the set of irreflexive resp. symmetric relations on X:

$$\mathfrak{irrefl}(X) := \overline{\mathsf{L};(\mathsf{M} \cap \mathrm{vec}(\mathsf{I});\mathsf{L})}^{\mathsf{T}} \qquad \mathfrak{symm}(X) := \overline{\mathsf{L};(\mathsf{M} \cap \overline{[\rho, \pi]\!];\mathsf{M}})}^{\mathsf{T}}$$

Example: Functions and Homomorphisms

Assume $s : X^2 \leftrightarrow \mathbf{1}$ to be given. Then we have:

$$
\begin{array}{rcll}
\mathrm{Rel}(s) \text{ univalent} & \iff & \mathrm{Rel}(s)^{\mathsf{T}};\mathrm{Rel}(s) \subseteq \mathsf{I} & \\
& \iff & \mathrm{Rel}(s);\overline{\mathsf{I}} \subseteq \overline{\mathrm{Rel}(s)} & \text{Schröder} \\
& \iff & \mathrm{vec}(\mathrm{Rel}(s);\overline{\mathsf{I}}) \subseteq \mathrm{vec}(\overline{\mathrm{Rel}(s)}) & \text{isom.} \\
& \iff & \mathrm{vec}(\mathsf{I};\mathrm{Rel}(s);\overline{\mathsf{I}}) \subseteq \overline{\mathrm{vec}(\mathrm{Rel}(s))} & \text{isom.} \\
& \iff & (\mathsf{I} \,\|\, \overline{\mathsf{I}});\mathrm{vec}(\mathrm{Rel}(s)) \subseteq \overline{\mathrm{vec}(\mathrm{Rel}(s))} & \text{Theorem 2} \\
& \iff & (\mathsf{I} \,\|\, \overline{\mathsf{I}});s \subseteq \overline{s} & \text{isom.}
\end{array}
$$

From Theorem 1 we get the relational specification

$$
\mathfrak{unival}(X, Y) := \overline{\mathsf{L};((\mathsf{I} \,\|\, \overline{\mathsf{I}});\mathbf{M} \cap \mathbf{M})}^{\mathsf{T}}
$$

of a vector $\mathfrak{unival}(X, Y) : [X \leftrightarrow Y] \leftrightarrow \mathbf{1}$ that models the set of univalent relations of type $X \leftrightarrow Y$.

Similarly Theorem 2 yields (π is the first projection relation of $X \times Y$)

$$\mathrm{Rel}(s) \text{ total} \iff \mathsf{L} \subseteq \pi^{\mathsf{T}};s.$$

From Theorem 1 we get the following vector $\mathfrak{total}(X, Y) : [X \leftrightarrow Y] \leftrightarrow \mathbf{1}$ that models the set of total relations of type $X \leftrightarrow Y$:

$$\mathfrak{total}(X, Y) := \overline{\mathsf{L};\overline{\pi^{\mathsf{T}};\mathbf{M}}}^{\mathsf{T}}$$

Intersection yields the following vector $\mathfrak{function}(X, Y) : [X \leftrightarrow Y] \leftrightarrow \mathbf{1}$ that models the set of functions type $X \leftrightarrow Y$:

$$\mathfrak{function}(X, Y) := \mathfrak{unival}(X, Y) \cap \mathfrak{total}(X, Y)$$

Given relations $R : X \leftrightarrow X$ and $S : Y \leftrightarrow Y$ a relation $F : X \leftrightarrow Y$ is a **homomorphism** from R to S if it is a function and $R;F \subseteq F;S$.

Assume R, S and $s : X \times Y \leftrightarrow \mathbf{1}$ to be given. Then we have:

$$\begin{aligned}
& R;\mathrm{Rel}(s) \subseteq \mathrm{Rel}(s);S \\
\iff\ & \mathrm{vec}(R;\mathrm{Rel}(s);\mathsf{I}) \subseteq \mathrm{vec}(\mathsf{I};\mathrm{Rel}(s);S) && \text{isom.} \\
\iff\ & (R \parallel \mathsf{I});\mathrm{vec}(\mathrm{Rel}(s)) \subseteq (\mathsf{I} \parallel S^{\mathsf{T}});\mathrm{vec}(\mathrm{Rel}(s)) && \text{Theorem 2} \\
\iff\ & (R \parallel \mathsf{I});s \subseteq (\mathsf{I} \parallel S^{\mathsf{T}});s && \text{isom.}
\end{aligned}$$

Combining Theorem 1 with the vector $\mathfrak{function}(X, Y)$ we get

$$\mathfrak{hom}(R, S) := \mathfrak{function}(X, Y) \cap \overline{\mathsf{L};((R \parallel \mathsf{I});\mathbf{M} \cap \overline{(\mathsf{I} \parallel S^{\mathsf{T}});\mathbf{M}})}^{\mathsf{T}}$$

as relational specification of a of a vector $\mathfrak{hom}(R, S) : [X \leftrightarrow Y] \leftrightarrow \mathbf{1}$ that models the set of homomorphisms from R to S.

An Application

In 1969 D. Scott presented a complete lattice $(D, \leq)$ that is isomorphic to the complete lattice $([D \to D], \sqsubseteq)$ of continuous functions on D.

$(D, \leq)$ is the inverse limit of a **retraction sequence** $(X_n, \leq_n, \varphi_n, \psi_n)_{n \in \mathbb{N}}$ of complete lattices, where:

- $X_0 := \{\bot, \top\}$ and $\bot \leq_0 \top$.
- $X_{n+1} := (D \to D)$ and $\leq_{n+1} :=$ function order induced by $\leq_n$.

In a textbook J. Stoy

- describes the construction in detail,
- presents $(X_1, \leq_1)$ and $(X_2, \leq_2)$,
- states that $|X_3| = 120\,549$.

We have used RelView

- to verify this number
- even to compute $\leq_3$ with $1\,805\,247\,020$ pairs as a ROBDD with $2\,500\,126$ nodes.

Decisive for that is the following relation-algebraic specification of the **function order**.

We need the **injective embedding** $\mathrm{inj}(v)$ induced by a vector $v : X \leftrightarrow \mathbf{1}$. It is the identity function from V to X, where V is the set modeled by v.

Theorem 5 Let $R : X \leftrightarrow X$ be a partial order, $\mathfrak{hom}(R, R) : [X{\leftrightarrow}X] \leftrightarrow \mathbf{1}$ be the vector that models the set $(X \to X)$ of R-monotone functions on X and $\mathsf{F} : (X \to X) \leftrightarrow (X \to X)$ be the function order on $(X \to X)$ induced by R. Then we have:

$$\mathsf{F} = \overline{\mathbf{N}^{\mathsf{T}};(R \parallel \overline{R});\mathbf{N}} \text{ where } \mathbf{N} := \mathbf{M};\mathrm{inj}(\mathfrak{hom}(R, R))^{\mathsf{T}}.$$

Relation $\mathbf{N} : X^2 \leftrightarrow (X \to X)$ relates u and f iff f_{u_1,u_2}, i.e., iff $f(u_1) = u_2$.

The proof: Assume arbitrary $f, g \in (X \to X)$. Then we have (where u and v range over X^2):

$$
\begin{aligned}
\mathsf{F}_{f,g} &\iff \forall u, v : f(u_1) = u_2 \land g(v_1) = v_2 \land R_{u_1,v_1} \Rightarrow R_{u_2,v_2} \\
&\iff \forall u, v : f_{u_1,u_2} \land g_{v_1,v_2} \land R_{u_1,v_1} \Rightarrow R_{u_2,v_2} \\
&\iff \neg\exists u, v : f_{u_1,u_2} \land g_{v_1,v_2} \land R_{u_1,v_1} \land \overline{R}_{u_2,v_2} \\
&\iff \neg\exists u, v : f_{u_1,u_2} \land (R \,\|\, \overline{R})_{u,v} \land g_{v_1,v_2} \\
&\iff \neg\exists u : \mathbf{N}_{u,f} \land \exists\, v : (R \,\|\, \overline{R})_{u,v} \land \mathbf{N}_{v,g} \\
&\iff \neg\exists u : \mathbf{N}^{\mathsf{T}}_{f,u} \land ((R \,\|\, \overline{R});\mathbf{N})_{u,g} \\
&\iff \overline{\mathbf{N};(R \,\|\, \overline{R});\mathbf{N}^{\mathsf{T}}}_{f,g}
\end{aligned}
$$

Hence, $\mathsf{F} = \overline{\mathbf{N}^{\mathsf{T}};(R \,\|\, \overline{R});\mathbf{N}}$.

Conclusion

RelView supports work with relations in a number of ways.

But the tool **only is able to treat set-theoretic relations on finite carrier sets**.

For this reason **results of experiments that seem to confirm an abstract relation-algebraic property are to handle with some care**.

- It may happen that the considered property holds for all set-theoretic relations but not in abstract relation algebras.
- It may happen that the considered property holds for all finite set-theoretic relations but not for infinite ones.

But, fortunately, such situations are rare.